Lecture 26 Summary

PHYS798S Spring 2016
Prof. Steven Anlage

May 9, 2016

1 Fluctuation Conductivity Above T_c

The resistance vs. temperature curves show precursor effects as the transition temperature is approached from above. The resistance is observed to drop at temperatures above T_c, to a degree that depends on the dimensionality of the material (0D, 1D, 2D, 3D). We treat the conductivity above T_c as a linear combination of mean-field conductivity (entirely due to normal state physics) and fluctuation conductivity (due to superconducting fluctuations):
$\sigma = \sigma_{MF} + \sigma^{fluc}$.
The idea is that the material borrows $k_B T$ of energy from the thermal bath temporarily and creates a superconducting fluctuation of limited size and limited lifetime somewhere in the material. While this fluctuation exists it creates a small zero-resistance pathway for current, leading to a drop in global resistance of the sample. These fluctuations occur rapidly and in many locations around the sample, leading to clearly measurable effects when measured on the time scales of seconds.

We will work with the Fourier transform of $\langle|\psi|^2\rangle$. Consider the GL free energy expansion above T_c ($\alpha > 0$), where the fluctuating order parameter is expected to be small. In this case we can ignore the $|\psi|^4$ term, and we will take the vector potential and external fields to be zero. This leaves a free energy expansion of,
$f = \alpha|\psi|^2 + \frac{\hbar^2}{2m^*}(\nabla\psi)^2$.
Now take the Fourier transform of ψ,
$\psi(\overrightarrow{r}) = \sum_k \psi_k e^{i\overrightarrow{k}\cdot\overrightarrow{r}}$. This gives for the free energy difference of the sample,
$F = \int f dV = \sum_k \left\{\alpha + \frac{\hbar^2 k^2}{2m^*}\right\}|\psi_k|^2$.

Now assume equipartition of energy to each degree-of-freedom of the fluctuating order parameter. This means that every term in the sum on k will have on average $k_B T$ of energy, or in other words,
$\langle|\psi_k|^2\rangle = \frac{k_B T/\alpha}{1+k^2\xi_{GL}^2}\frac{1}{V}$, where we have used the fact that $\xi_{GL}^2 = \frac{\hbar^2}{2m^*\alpha}$.
The k-dependence suppresses the short wavelength fluctuations. In fact we should introduce a cutoff in the k-sum because wavenumbers larger than $1/\xi_{GL}$

cannot be expected to contribute.

How big are the areas that fluctuate in to the superconducting state above T_c? Below T_c the order parameter is assumed to be long-range phase coherent. Above T_c one can calculate the two-point correlation function $g(\overrightarrow{r}, \overrightarrow{r}') \equiv \langle \psi^*(\overrightarrow{r})\psi(\overrightarrow{r}')\rangle$, which is found to be $g(\overrightarrow{r}, \overrightarrow{r}') = \frac{m^* k_B T}{2\pi\hbar^2} \frac{e^{-R/\xi_{GL}(T)}}{R}$, where $R = |\overrightarrow{r} - \overrightarrow{r}'|$ (see Tinkham p. 300). The typical size of the fluctuation is on the order of the GL coherence length above T_c, with $\xi_{GL}^2 = \frac{\hbar^2}{2m^*\alpha}$ and $\alpha > 0$.

2 Brief Introduction to Time-Dependent Ginzburg-Landau Theory (TDGL)

Above T_c the equilibrium value of the order parameter is zero, $\psi_0 = 0$. Any fluctuation results in a non-zero order parameter, as we assume $\psi = \psi_0 + \delta\psi$. TDGL says that such a fluctuation will relax back to zero exponentially in time: $\frac{d\psi}{dt} = -\frac{1}{\tau_0}\psi$. The result for the $k = 0$ momentum relaxation time is $\tau_0 = \frac{\pi\hbar}{8k_B(T-T_c)} = \frac{3ps-K}{(T-T_c)}$. (This result is derived for gapless superconductors by M. Cyrot, Rep. Prog. Phys. 36, 103 (1973).)
Now consider the GL equation corresponding to the free energy expansion given above,
$\alpha\psi - \frac{\hbar^2}{2m^*} \bigtriangledown^2 \psi = 0$, or after dividing through by α,
$\xi_{GL}^2 \bigtriangledown^2 \psi - \psi = 0$. The TDGL generalization of this equation is,
$\frac{d\psi}{dt} = -\frac{1}{\tau_0}(1 - \xi_{GL}^2\bigtriangledown^2)\psi$.
Generalize the Fourier transform of the order parameter to a time-dependent version,
$\psi(\overrightarrow{r}, t) = \sum_k \psi_k e^{i\overrightarrow{k}\cdot\overrightarrow{r}} e^{-t/\tau_k}$. Putting this into the generalized TDGL equation yields,
$\tau_k = \frac{\tau_0}{1+k^2\xi_{GL}^2}$. This shows that short wavelength fluctuations will decay more quickly.

3 Fluctuation Conductivity in TDGL

Tinkham derives the fluctuation conductivity above T_c using the Kubo formalism. Here we use a simple TDGL argument. In analogy with the Drude expression for the mean field conductivity, $\sigma_n = \frac{ne^2\tau}{m}$, let's try,
$\sigma^{fluc} = \frac{(e^*)^2}{m^*} \sum_k^{k_{cutoff}} \langle|\psi_k|^2\rangle \frac{\tau_k}{2}$, where the factor of 2 comes from the fact that ψ_k^2 decays twice as fast as ψ_k.
Substituting in the results above yields,
$\sigma^{fluc} = \frac{(e^*)^2}{\hbar^2} \frac{\xi_{GL}^2}{V} k_B T \tau_0 \sum_k^{k_{cutoff}} \frac{1}{(1+\xi_{GL}^2 k^2)^2}$. The outcome of the sum (turned in to an integral) depends on the dimensionality d of the system.

The results are,

$d = 3$: $\sigma_{3D}^{fluc} = \frac{e^2}{\hbar \xi_{GL}(0)} (\frac{T}{T-T_c})^{1/2} \times \begin{cases} 1/4 & \text{This approximation} \\ 1/32 & \text{Kubo} \end{cases}$

where we write $\xi_{GL}(T) = \xi_{GL}(0)\sqrt{\frac{T}{T-T_c}}$.

$d = 2$: $\sigma_{2D}^{fluc} = \frac{e^2}{\hbar t} (\frac{T}{T-T_c}) \times \begin{cases} 1/8 & \text{This approximation} \\ 1/16 & \text{Kubo} \end{cases}$

where t is the film thickness.

$d = 1$: $\sigma_{1D}^{fluc} = \frac{e^2}{\hbar} (\frac{T}{T-T_c})^{3/2} \frac{\xi_{GL}(0)}{A_c} \times \begin{cases} 1/8 & \text{This approximation} \\ \pi/16 & \text{Kubo} \end{cases}$

where A_c is the cross sectional area of the 1D wire.

Note that in all cases the scale of the fluctuation conductivity is set by the quantum of conductance $\frac{e^2}{\hbar} = 243.3 \mu S$. Alternatively one can write $\frac{\hbar}{e^2} = 4.11 k\Omega$.
The dc fluctuation conductivity diverges at T_c more strongly in lower dimensionality as
$\sigma_{dD}^{fluc} \propto \frac{1}{(T-T_c)^{(4-d)/2}}$.
The class web site shows fluctuation conductivity data from 1 and 2-dimensional materials.

4 The Kosterlitz-Thouless Phase Transition for 2D Superconductors

One does not have true long-range order (LRO) in 1 or 2 dimensions. The 3D BCS ground state is a coherent state of Cooper pairs that maintains phase coherence over effectively infinite distance. In lower dimensions, the phase coherence is less strong. This is quantified by the two-point correlation function for the GL order parameter. In 3D one has $\langle \psi^*(\overrightarrow{r})\psi(\overrightarrow{r}')\rangle \sim$ constant as $|\overrightarrow{R}| = |\overrightarrow{r} - \overrightarrow{r}'| \to \infty$. Above T_c one finds,
$\langle \psi^*(\overrightarrow{r})\psi(\overrightarrow{r}')\rangle \sim e^{-R/\xi_N}$.
In 2D at low temperature one finds, $\langle \psi^*(\overrightarrow{r})\psi(\overrightarrow{r}')\rangle \sim \frac{1}{R^{\eta(T)}}$. This difference between the exponential decay above T_c to a power-law decay at low temperatures implies a phase transition. This is the famous "KT" transition that is seen in 2D Coulomb gases and in vortices in thin films of superfluid ^4He. The original reference is J. Phys. C 6, 181 (1973).

The theory examines the lowest energy excitations out of the ground state. Individual vortices have an energy that scales logarithmically with the size of the sample. Bound Vortex/Anti-vortex (V/AV) pairs have a much smaller energy, hence they dominate the low temperature properties. As temperature increases, there is an entropic advantage to de-pairing the V/AV pairs and creating unbound free vortices. This precipitates the KT transition at T_{KT}. Below T_{KT} there is zero resistance in the limit of current going to zero. In other words the critical current is zero! Above T_{KT} there is finite resistance even in the limit as the current goes to zero.

5 KT in 2D Superconductors: Vortex and V/AV Energies

Vortices in 2D superconductors are similar to those discussed before in 3D superconductors except for the tails. Instead of having the currents falling off exponentially with distance for $r > \lambda$ in 3D, one instead has a surface current given by,

$$\overrightarrow{K}_s(r) = \widehat{\theta} \times \begin{cases} \frac{\Phi_0}{2\pi}\frac{d/\lambda^2}{r} & r << 2\lambda^2/d \\ \frac{\Phi_0}{2\pi}\frac{2}{r^2} & r >> 2\lambda^2/d \end{cases}$$

See the paper by Pearl, Appl. Phys. Lett. 5, 65 (1964). The key things to note are the $1/r^2$ drop-off of the surface currents with distance, and the crossover length scale, called the perpendicular penetration depth $\lambda_\perp = 2\lambda^2/d$, where d is the film thickness. The crossover length scale can be macroscopic in size in low carrier density and/or disordered superconducting films of small (nm) thickness. Thus the $1/r$ "core" of the vortices can extend over macroscopic distances! The vortices now act like Coulomb charges interacting in a 2D metal.

The energy of a free vortex can be calculated by ignoring the vortex core (GL $\kappa \to \infty$) and considering only the kinetic energy of the currents as, $W_1 = \pi n^*_{s,2D}\frac{\hbar^2}{m^*}\ln\frac{R}{r_0}$, where $n^*_{s,2D} = n_s L$ is the 2D superfluid density, n_s is the 3D superfluid density, L is the length of the vortex (on the order of the film thickness), r_0 is the microscopic length scale where the current density approaches the de-pairing value (we expect $r_0 \sim \xi_{GL}$), and R is the sample size, where it is assumed that $\lambda_\perp$ is much greater than the sample size. The energy of a single isolated vortex scales with the system size, making it very expensive!

Contrast this with the case of a V/AV pair at some distance r apart. Far away ($R >> r$) the flow fields of the two vortices cancel to good approximation, making the object appear "neutral" from far away. The currents are strong only within r, giving rise to a total energy of just,
$W_2 = 2\pi n^*_{s,2D}\frac{\hbar^2}{m^*}\ln\frac{r}{r_0}$.
Because $W_2 << W_1$ the V/AV excitations are the dominant excitations at low temperature in the 2D superconductor.